Journal of Physics Special Topics

P5_1 Cinderella's Shattered Dreams

H. Samaratunga, L. Tonks, H. S. Ahmed and M. Sadhra

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH.

October 28, 2015

Abstract

Cinderella famously went to the ball in glass shoes; however, the practicality of this choice of footwear remains to be seen. The heel height of these glass shoes that could safely be used was determined for; standing, walking and running in them. While the glass heel can be any height and not break while standing, the height of the heel while walking was between 3.44cm and 13.8cm and running was between 1.15cm and 4.61cm. As Cinderella ran away from the prince at midnight the height the heel can safely be is 1.15cm; this is much smaller than is often depicted.

Introduction

As portrayed in many different versions of the fairy tale Cinderella, Cinderella is given a pair of glass shoes by her fairy godmother. Cinderella then goes on to attend a ball in these glass shoes, a ball at which she famously runs away from the prince at midnight. We will assume that the type of glass used to make the shoe is soda lime glass; this is an everyday glass. Given that glass is brittle it therefore does not undergo plastic deformation and the glass shoe will break once a certain force has been reached. Outlined in this paper are the various actions that Cinderella would be undertaking in these shoes, namely; standing, walking and running and if they would be possible without the glass shoe breaking, also addressed is the maximum heel height of the glass shoe that could safely be used for the various motions.

Discussion

To determine Cinderella's ability to move in her glass shoes we started with addressing her ability to stand in them using the stress equation

$$\sigma = \frac{F}{A} \quad (1)$$

where F is the force acting downwards i.e. her weight and A is the surface area of her foot. In order to calculate, a few assumptions were made about her weight and foot size. Cinderella weighs 55kg and the foot size we chose is a UK size 4. In the story Cinderella has smaller feet than her stepsisters therefore we presumed if the average foot size is a size 6, a size 4 would be a reasonable estimation for her.

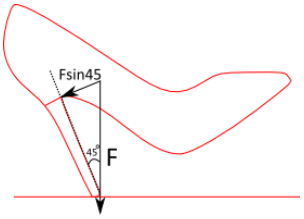

Figure 1: Motion of the glass shoe when walking or running (not to scale)

When standing the force acting downward is assumed to be evenly shared between her feet, therefore for one shoe the weight is halved and the surface area of her foot is approximated to be 0.0095 m^2 [1]. Using equation 1, if 27.5kgs is applied uniformly across this area, the compressive stress (σ) developed in the material would be 28.4kPa. The type of glass we have assumed, Soda Lime, has a compressive strength before rupture of 330MPa [2] which is considerably much larger than what we calculate. Therefore Cinderella will be able to stand still in her glass shoes without fear of them breaking.

However, if Cinderella wants to get to her ball she will have to be able to walk and ultimately run in these shoes. When walking or running, the action is assumed to require that all the weight is on one foot at a time, and that her motion is heel first. We have assumed a 45-degree angle when making these steps. The heel then modelled as a cantilever beam with a square cross-section of 1cm by 1cm, this is done to simplify the problem and helps calculate the bending stress. The equation for bending stress, σ_b [3]:

$$\sigma_b = \frac{F_2 Lc}{J} \qquad (2)$$

The force applied in the normal direction of the heel, F_2, (where F_2 = Fsin45) and this force is what causes the bending stress in the heel (see *figure 1*). The c term is the perpendicular distance to the natural axis of the heel, which 0.5cm and L is the height of the heel which is our variable. J is the moment of inertia of a plane area about a neutral axis also known as the second moment of inertia [3]; this is found using this equation for a rectangle:

$$J = \frac{bh}{12}(b^2 + h^2) \qquad (3)$$

The base (b) and the height (h) of the beams cross section, in our case are the same, 1cm, therefore the second moment of inertia was calculated to be $1.6\text{x}10^{-9}\text{m}^4$. By applying a range of values for the heel height, L, from 0.5cm to 15cm and using equation 2 to calculate the bending stress ay each height we plotted a graph showing the relationship between the two (*see figure 2*).

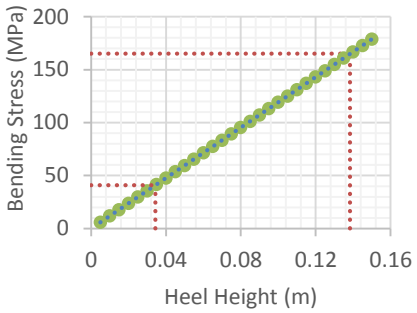

Figure 2 bending stress when walking relative to the heel height

The given Soda Lime glass' flexural strength ranges between 42-165MPa [2], this is the maximum flexural stress to rupture and it is determined by a standardized flexural (or bending) test. Figure 2 indicates the flexural strength of Soda Lime glass, (red dotted lines). This gives the minimum and maximum heel height possible for Cinderella to walk at 3.44cm and 13.8cm respectively.

As for running the force due to the rapid high impact can be thrice that of the body weight [4], thus giving a new force normal to the heel, F_2. Using the same method as before and plotting the graph (*figure 3*). The minimum and maximum heel height that can be used when running before rupture is 1.15cm and 4.61cm, for Soda Lime glass.

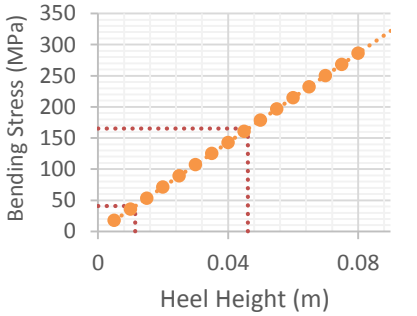

Figure 3 bending stress due to running relative to heel height

Conclusion

While Cinderella would be able to stand in her shoes regardless of the size of the heel the same cannot be said for walking and running. Being conservative and using the lowest value for the flexural stress to rupture for safety reasons the results show that in order for Cinderella to run away from the prince at midnight she would have to have a glass shoe with a heel of less than 1.15cm. This is much smaller than the size of heel that is often portrayed in adaptations of the fairy tale. These results have been calculated with certain assumptions; to further improve the estimate of the height of the heel, the results for different running/walking angle of impact as well as different types of glass should be determined as these would also affect how high the heel could be before breaking.

References

[1]http://www.p2d.co.uk/acatalog/Shoe_Size_Conversion_Chart.html /accessed on 04/10/2015

[2]http://www.makeitfrom.com/material-properties/Soda-Lime-Float-Glass /accessed on 04/10/2015

[3]http://courses.washington.edu/biomechs/lectures/lecture12.pdft /accessed on 05/10/2015

[4]http://www.barefootrunning.fas.harvard.edu/4BiomechanicsofFootStrike.html /accessed on 05/10/2015