Demonstrating Kepler's Laws using orbital data of Mercury, Saturn, and the Solar System

The purpose of this exercise is to verify Kepler's Laws – or discover them for yourself - using data collected by someone *else*, without having to spend months and years doing the observations yourself.

Johannes Kepler took over as Royal Danish Astronomer from Tycho Brahe around 1601, when Tycho suddenly died of an infection. Tycho had spent his life creating a most amazing observatory on the island of Hven, off the coast of Denmark, without a telescope – which was not invented until after his death. He was able to collect massive amounts of data on planetary orbits, but it was Kepler who, after Tycho's death, organized the patterns he observed in Tycho's data into what have become known as Kepler's Laws of planetary motion.

Later, Newton was able to explain the theoretical basis for Kepler's observations in terms of his laws of motion and universal law of gravity, and we now know that even Newton's Laws have a deeper explanation, based on Conservation of Energy and Conservation of Angular Momentum.

In this lab exercise you will:

1. Plot the distance of Mercury from the Sun in Astronomical Units (AU) for equal time intervals, to derive the shape of Mercury's orbit;
2. Measure the area swept out by radius vectors at perihelion (closest approach) and aphelion (farthest approach) and demonstrate that the areas swept out in equal time periods are equal;
3. Use the distance from the Sun and orbital period data for the solar system as a whole to derive the relationship between the period and semi-major axis of any planet.
4. In conclusion, you should have a feeling for what Kepler's Laws mean. In class we will derive them in terms of conservation of energy and angular momentum.

Please do not skip steps!!! thanks…

Equipment:

Graph paper
Protractor with Ruler
Laptop for graphing in Excel

Procedure:

1: Given the distance from the Sun of 18 positions of the planet Mercury, plot the orbit of Mercury around the Sun.

Background: Mercury makes one complete orbit around the Sun in 88 days. We have 18 observations, so that means that 5 days separate each observation, except the last one, which is only a 3-day interval. The distance measurements are measured from the center of the Sun to the center of Mercury, and the units are "AU" for Astronomical Units. One AU is the average distance between the Sun and the Earth, which is equal to approximately 1.5×10^8 km (150,000,000 km).

<u>Data:</u> The degrees indicate the angular distance that Mercury has traveled for each time interval, and the distances are measured from the Sun's center of mass to Mercury's center of mass.

observation #	Δt	degrees	distance from Sun (AU)
1	5	4	.35
2	10	31	.32
3	15	61	.31
4	20	92	.31
5	25	122	.32
6	30	149	.35
7	35	172	.38
8	40	192	.41
9	45	209	.43
10	50	224	.45
11	55	239	.46
12	60	252	.47
13	65	266	.47
14	70	280	.46
15	75	295	.44
16	80	311	.42
17	85	330	.40
18	88	350	.37

1. Holding a piece of graph paper sideways (landscape), put a dot for the Sun a distance about 5 inches (in the center of the paper) from the bottom. (This choice of where to put one focus is so that your ellipse will fit on the paper.)

2. Plot the positions of Mercury relative to the Sun, according to the data table. Start with 0^0 on the right, and **go counter clockwise.** Use a scale of 1 cm = 0.1 Au.

3. Connect the dots neatly and smoothly, and – voila! – what shape is the orbit? ____________________

****Which of Kepler's Laws have you demonstrated? Look it up in your text, on the internet, or ask your awesome lab instructor, and write it down:***

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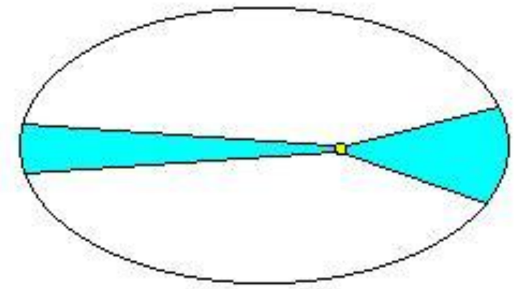
<u>2:</u> Verify Kepler's Second Law using the graph and data from Part 1.

1. Select two points that are closest to the Sun (I suggest #'s 3 and 4) and two that are farthest from the sun (I suggest #'s 12 and 13), as suggested in the figure to the right.



2. Draw the radius vector from the Sun to each data point. Call the short, thick sector "A" and the long, thin sector "B" (or whatever label you want to give them).

3.. Label the distance in AU to each data point. Find the average radius for each sector as

$r_{average} = (r_1 + r_2)/2$, and measure the central angle, Θ, for each sector.

4. Compute the approximate area of each sector as: $(\Theta/360) \times \pi (r_{average})^2$

Area of A = _______________________ Area of B = _____________________________ in units of AU^2

Questions:

a. Are the areas swept out by a radius vector in equal time intervals equal, at least within your experimental error?

b. Explain which of Kepler's Laws you have demonstrated.

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c. When is Mercury moving faster, when it is closest or farthest from the Sun? ______________

d. Explain this in terms of: ______________________

i. Conservation of Energy (gravitational potential and kinetic):

ii. Conservation of Angular Momentum:

<u>3:</u> Period – distance data, or Kepler's Law of Harmonies

Using data for the solar system as a whole, you will demonstrate Kepler's Third Law, which he called his Law of Harmonies.

Here are the data for semi-major axis of the orbit of each planet relative to the Sun in AU, and their orbital periods in years:

Planet	semi-major axis (AU)	Period (years)
Mercury	.386	.241
Venus	.720	.615
Earth	1.00	1.00
Mars	1.52	1.88
Jupiter	5.19	11.9
Saturn	9.53	29.5
Uranus	19.1	84
Neptune	30.0	165
Pluto	39.3	248

Note: Print out these graphs. Make sure to identify their shapes. You may put them on one page.

1. Copy these data into Excel. Graph the length of the semi-major axis in AU on the y-axis, and the orbital period in years on the x-axis. ***Print the graph. What does this graph look***?
What type of graph is this?

2. Use Excel to calculate the cube of the semi-major axis (a^3) and the square of the period (p^2) and graph a^3 vs. p^2. Suppose you're a-values are in column B, starting in row 2, for example. To calculate a^3 you input the following into column D (for example): "=B2^3" and then drag the lower right corner down as far as you want this formula to be copied. Voila! you have calculated the cube of the semi-major axis for the orbit of each planet. Repeat this process to get the square of the period. ***Print the graph. What does this graph look***?
What type of graph is this?

3. 3. Now use Excel to calculate the log of a^3 and log of p^2. Just use the log function. Suppose your a^3 data are in column D, and you want to put the log values in column F. In F2 you type "=log(D2,10)" for log of the value that is in D2, base 10. Drag the corner of this cell down as far as you want the calculations, and voila! – you have the log of the cube of the semi-major axes for all the planets. Do the same for the log of the square of the periods. ***Print the graph. What does this graph look***?
What type of graph is this?

Now state Kepler's Third Law in your own words: